HYDRODYNAMICS OF A CENTRIFUGAL SPRAYER

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An asymptotic analysis is made of the flow of a film of an inviscid incompressible liquid in a centrifugal sprayer under high heads. The conditions under which the film turns through an angle $\sim\pi$ along the surface of the sprayer are determined.

The swirling of streams of liquids and gases is widely used in industry as an effective means of intensifying heat- and mass-exchange processes. A centrifugal sprayer is an example of a technical device in which stream swirling is used [1, 2]. In such a device the swirled stream usually turns through an angle $\sim\pi/2$. The case when a swirled high-head stream turns through an angle $\sim\pi$ is studied below. The investigation is made under the following assumptions. The liquid is inviscid, incompressible, and weightless. The thickness of the liquid layer in the inner channel of the centrifugal sprayer is far less than its radius, while the velocity along the generating line of the channel is far lower than the velocity of rotation of the liquid. The total pressure of the liquid is far higher than the static pressure in the surrounding space. In this case, the following characteristic regions of flow can be distinguished (Fig. 1): region I ($\infty > y > 0$) of rotational—translational flow of the liquid film in the inner channel; region II, in which the liquid film turns through an angle $\pi/2$ along the surface of the sprayer in the vicinity of ($y = 0$, $r = r_0$); in region III, located on the outer surface, the liquid film turns through another angle $\sim\pi/2$, so that the flow turns through a total angle $\sim\pi$. The conditions under which the flow of the liquid film proves to be attached are studied in the paper. This is important from a practical point of view, since a detached film rapidly loses stability and is broken up by forces of surface tension. As a result of the loss of continuity of the liquid film, the surface of the sprayer is not protected from the adverse action of the external medium or overheating.

The ability of a stream of liquid or gas to bend around a surface is called the Coanda effect. Thus, a theory of the Coanda effect for a centrifugal sprayer is presented here.

1. We consider the rotational—translational flow of a layer of an inviscid incompressible liquid along the inner cylindrical channel, of radius r_0, of a sprayer (Fig. 1). Let the thickness of the liquid film be $\delta \ll r_0$, while the velocity component along the generating line is $u_0 \ll w_0$, where w_0 is the rotational component of the velocity vector. In the cross section $y = 0$, where y is the axis of symmetry of the sprayer, the film turns through an angle $\sim\pi/2$. Obviously, such flow is not always possible. If the edge is sharp, the liquid film detaches from the surface and breaks up rather rapidly. Let us try to estimate the characteristic size and shape of the sprayer surface in the vicinity of $y = 0$ and $r = r_0$ (Fig. 2) providing for the attached turning of the liquid film through an angle $\sim\pi/2$. If we neglect the variation of the film thickness δ as it spreads out, then the following forces act on an element $\Delta l\delta\Delta\varphi$ of this film with a mass $m = \Delta l\delta\Delta\varphi\rho$:

$$F_1 \sin\theta + F_2 + \Delta p r \Delta\varphi \Delta l = 0. \quad (1)$$

Here $\Delta\varphi$ is an element of azimuth angle; Δp is the pressure difference between the outer and inner surfaces of the liquid; F_1 and F_2 are the centrifugal forces determined by the rotational and translational motion of the liquid. We shall neglect the term allowing for the pressure difference Δp, since this force promotes the pressing of the film against the surface $y = y(r)$ along which it moves, and the resulting estimate will be majorant for the attached surface of the sprayer.

The values of the centrifugal force depend on the rotational motion,

$$F_1 \approx m w_0^2 / r, \quad (2)$$

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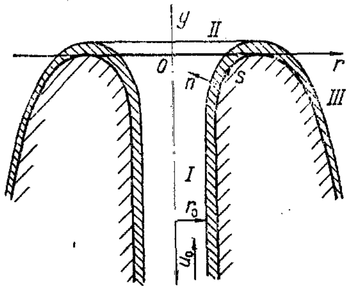

Fig. 1

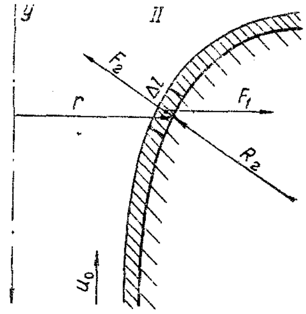

Fig. 2

Fig. 1. Diagram of liquid flow in a centrifugal sprayer.

Fig. 2. Diagram of liquid flow in region II.

and the longitudinal velocity,

$$F_2 \approx mu_0^2/R. \tag{3}$$

In accordance with the definition of the radius of curvature R of the surface y(r).

$$\frac{1}{R} = \pm \frac{d^2y}{dr^2}\left[1+\left(\frac{dy}{dr}\right)^2\right]^{-3/2}. \tag{4}$$

From Eqs. (1)-(4), we get

$$\frac{w_0^2}{r}\frac{dy}{dr} + u_0^2\frac{d^2y}{dr^2}\left[1+\left(\frac{dy}{dr}\right)^2\right]^{-1} = 0. \tag{5}$$

We require that the two terms of Eq. (5) be of the same order. Then we obtain

$$\Delta y \approx \Delta r, \quad r \simeq r_0 + \Delta r, \quad w_0 \approx u_0/\sqrt{\Delta r/r_0}. \tag{6}$$

We designate $A = w_0^2\Delta r/u_0^2 r_0 = O(1)$, $\Delta r = l_0$, $r = r_0 + l_0 x_1$, $y = l_0 y_1$. Equation (5) takes the form

$$\frac{d^2y_1}{dx_1^2} + A\frac{dy_1}{dx_1}\left[1+\left(\frac{dy_1}{dx_1}\right)^2\right] = 0. \tag{7}$$

As the boundary conditions for Eq. (7), we use the requirement that the curve join smoothly with the face, i.e.,

$$\begin{aligned} y_1 &\to 0 \quad \text{as} \quad x_1 \to \infty, \\ dy_1/dx_1 &\to 0 \quad \text{as} \quad x_1 \to \infty. \end{aligned} \tag{8}$$

The solution of Eq. (7) under the conditions (8) is

$$y_1 = \pm \frac{L}{A} \arcsin\left[\exp\left(-Ax_1\right)\right]. \tag{9}$$

Under the condition of conjugation of (9), $y_1 = y_1(x_1)$, with the inner wall of the sprayer, we take the positive sign. The coordinate of the conjugation point is $y_1(0) = \pi/2A$.

Thus, for $w_0 \gg u_0$ we find that the turning of the liquid film will occur in the region of $\Delta r \approx \Delta y$ in the vicinity of ($y = 0$, $r = r_0$) along the surface (9). The scale of this region is defined as

$$\Delta r = l_0 \approx r_0 u_0^2/w_0^2. \tag{10}$$

Later, using the estimates (6) and (10), we shall carry out an asymptotic analysis of solutions of the Euler equations for $w_0/u_0 \to \infty$ and $A = O(1)$. Since the quantity A is finite and the solution of the equations depends on it, it is a similarity parameter of this flow.

Let us find the characteristic value of the transverse velocity component (normal to the surface) in the region of turning of the liquid film along a section of length l_0. From the continuity equation we can obtain $v \sim \delta u_0/l_0$.

One can imagine the following cases: 1) $\delta \ll l_0$; 2) $\delta \sim l_0$; 3) $\delta \gg l_0$. The last one will not be considered here, since at scales of δ the turning region contracts to a corner point and the estimate for v proves to be wrong. Detachment of the liquid film from the surface is the most probable in this case.

Let us consider the case of $\delta \ll l_0$. We introduce the coordinates

$$s = l_0 s_1, \quad n = \delta n_1 \tag{11}$$

and the asymptotic expressions

$$\begin{gathered} u = u_0 u_1 + \ldots, \; v = (\delta/l_0) u_0 v_1 + \ldots, \; w = w_0 w_1 + \ldots, \; p = p_0 p_1 + \ldots, \\ h_1 = 1 + \delta (n_1/R l_0) + \ldots, \; h_3 = r_0 + l_0 r_1 + \delta n_1 \cos\theta + \ldots . \end{gathered} \tag{12}$$

Here $h_1 = 1 + n/R$ and $h_3 = r + n \cos\theta$.

We substitute Eqs. (11) and (12) into the Euler equations, written in orthogonal coordinates (s, n, φ) connected with the surface of the sprayer, and we carry out the limiting transition $\delta \to 0$, $l_0 \to 0$, $\delta/r_0 \to 0$, $u_0/w_0 \to 0$, and $\delta/l_0 \to 0$. As a result, we obtain (omitting the subscript) the system in the first approximation:

$$\begin{gathered} \frac{\partial u}{\partial s} + \frac{\partial v}{\partial n} = 0, \quad \frac{\partial p}{\partial n} = 0, \quad u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial n} - w^2 \frac{dr}{ds} = -\mathrm{Eu} \frac{\partial p}{\partial s}, \\ u \frac{\partial w}{\partial s} + v \frac{\partial w}{\partial n} = 0. \end{gathered} \tag{13}$$

Here $\mathrm{Eu} = p_0/\rho u_0^2$. We note that a pressure gradient across the liquid layer is absent from the system of equations (13), and thus, there is no force capable of detaching the liquid from the surface. These equations are of the parabolic type and have the form of the equations of an inviscid boundary layer.

Let us proceed to the case of $\delta \sim l_0$. We assume that $l_0 = \alpha\delta$, where $\alpha = O(1)$. From this we obtain $u_0/w_0 = \sqrt{\alpha\delta/r_0}$.

We introduce the coordinates

$$s = l_0 s_2, \quad n = \delta n_2 \tag{14}$$

and the asymptotic expansions

$$\begin{gathered} u = u_0 u_2 + \ldots, \; v = (\delta/l_0) u_0 v_2 + \ldots, \; w = w_0 w_2 + \ldots, \; p = p_0 p_2 + \ldots, \\ h_1 = 1 + n_2/\alpha R_2 + \ldots, \; h_3 = r_0 + \delta(\alpha r_2 + n_2 \cos\theta) + \ldots . \end{gathered} \tag{15}$$

After the substitution of Eqs. (14) and (15) into the Euler equations and the limiting transition $\delta \to 0$, $l_0 \to 0$, $\delta/r_0 \to 0$, $u_0/w_0 \to 0$, $\delta/l_0 = O(1)$, $\alpha = O(1)$, we obtain the following system of equations in the first approximation (the subscript is omitted):

$$\begin{gathered} \frac{\partial u}{\partial s} + \frac{\partial}{\partial n}\left[\left(1 + \frac{n}{\alpha R}\right) v\right] = 0, \\ \frac{u}{1 + n/\alpha R} \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial n} + \frac{uv}{\alpha R} - \frac{w^2}{1 + n/\alpha R}\left[\frac{dr}{ds} + \frac{n}{\alpha}\frac{d}{ds}(\cos\theta)\right] = -\mathrm{Eu}\frac{\partial p}{\partial s}, \\ \frac{u}{1 + n/\alpha R} \frac{\partial v}{\partial s} + v \frac{\partial v}{\partial n} - \frac{\alpha u^2}{R(1 + n/\alpha R)} - \alpha w^2 \cos\theta = -\alpha^2 \mathrm{Eu} \frac{\partial p}{\partial n}, \\ u \frac{\partial w}{\partial s} + w\left(1 + \frac{n}{\alpha R}\right)\frac{\partial w}{\partial n} = 0. \end{gathered} \tag{16}$$

The boundary conditions for the systems of equations (13) and (16) have the usual form. They are the condition of nonpenetration at the solid surface and the assigned pressure distribution over the outer boundary of the film. The initial conditions for these systems of equations must be sought using joining principles. As a result, we can obtain

$$\lim_{s_i\to-\infty} u_i(s_i,\ n_i) = \lim_{s_0\to 0} u_0(s_0,\ n_0),$$
$$\lim_{s_i\to-\infty} w_i(s_i,\ n_i) = \lim_{s_0\to 0} w_0(s_0,\ n_0).$$

Here i = 1 or 2, while the index 0 corresponds to the rotational—translational flow of the liquid film in the inner channel of the sprayer [1].

2. Let us consider the flow of the liquid film on the outer surface of a centrifugal sprayer (region III, Fig. 1), assuming that the flow over the corner region II is attached. We shall assume that the characteristic linear size L of this surface and its radius of curvature R are of the same order of magnitude (Fig. 3). For a weightless liquid, the only force capable of curving the trajectory of motion of the liquid layer is a pressure drop between the wall (inner) streamline and the outer streamline, $\Delta p = p_W - p_0$. Here p_W is the pressure at the surface of the sprayer; p_0 is the characteristic pressure at the outer boundary of the film. The centrifugal forces tending to detach the liquid film will be balanced by this pressure drop:

$$\frac{\rho w_0^2 \delta \Delta\Sigma}{R} \backsim \Delta p \Delta\Sigma. \tag{17}$$

Here w_0 is the characteristic velocity of the liquid layer.

If the local pressure at the inner boundary of the film decreases to the saturation pressure of the vapor, the liquid will bubble up. Vapor bubbles appear and the liquid film "detaches" from the surface of the sprayer. We shall assume that $\Delta p/p_0 = O(1)$, and then from (17) we obtain an estimate of the radius of curvature R of the outer surface of a centrifugal sprayer,

$$R \backsim \delta/\mathrm{Eu}_1, \tag{18}$$

where $\mathrm{Eu}_1 = p_0/\rho w_0^2$ is the Euler numer; its definition differs from that of Eu introduced earlier.

From the relation (18) it follows that for radii of curvature of the outer surface of a sprayer of less than δ/Eu_1 in order of magnitude, the liquid film detaches from this surface, while for radii of curvature of the surface greater than δ/Eu_1 in order of magnitude, the flow will be attached, while the pressure drop across the stream will be small. The dimensionless complex $\lambda = (\delta/L)\mathrm{Eu}_1$, in addition to the Euler number Eu_1, the ratios δ/L and $\Delta p/p_0$, and the saturation pressure of the liquid, is a similarity parameter of such flows. Later we shall seek asymptotic solutions of the Euler equations as $\delta/L \to 0$ and $\mathrm{Eu}_1 \to 0$ and for $\lambda = O(1)$ and $\Delta p/p_0 = O(1)$.

The Euler equations are written, as before, in orthogonal variables (s, n, φ) connected with the surface of the sprayer.

The longitudinal size of the flow region III is $L \stackrel{\sim}{\sim} R$ and the transverse size is δ. In the general case L may also be far larger than R in the relation (18), in which case the pressure drop across the stream is small, in accordance with the estimate found. In the equations describing such flow, the terms containing pressure gradients will be negligibly small, since $\mathrm{Eu}_1 \ll 1$ and $L \gg R$. Such flows are characterized by the attached regime of flow of the liquid film over the surface. For practical use, it often becomes important to study the attached flow of a film in the presence of a transverse pressure gradient, i.e., the case of $\lambda = O(1)$.

Now let $L = \delta/\mathrm{Eu}_1$; we introduce the coordinates

$$s = Ls_3,\ n = \delta n_3 \tag{19}$$

and the asymptotic expansions

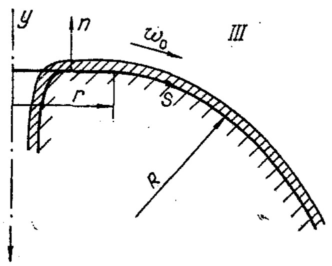

Fig. 3

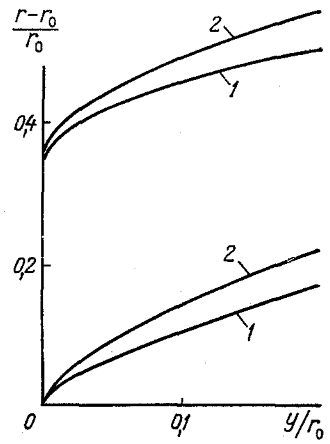

Fig. 4

Fig. 3. Diagram of liquid flow in region III.

Fig. 4. Outer surface of an "attached" sprayer (the upper curves correspond to the presence of a flat face of width Δr = 0.1 cm; for the lower curves, the flat face is absent, Δr = 0.0): 1) liquid flow rate m = 2; 2) 4 g/sec.

$$u = w_0 u_3 + \ldots, \; v = w_0 \, \mathrm{Eu}_1 \, v_3 + \ldots, \qquad w = w_0 w_3 + \ldots, \; p = p_0 p_3 + \ldots . \tag{20}$$

If we substitute Eqs. (19) and (20) into the Euler equations and carry out the limiting transition $\delta/L \to 0$, $\mathrm{Eu}_1 \to 0$, $\lambda = O(1)$, and $\Delta p/p_0 = O(1)$, we can obtain the following system of equations (the subscript 3 is omitted):

$$\frac{\partial (ru)}{\partial s} + \frac{\partial (rv)}{\partial n} = 0, \; u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial n} - \frac{w^2}{r}\frac{dr}{ds} = 0, \qquad \lambda\left(\frac{u^2}{R} + w^2\cos\theta\right) = \frac{\partial p}{\partial n}, \; u\frac{\partial w}{\partial s} + v\frac{\partial w}{\partial n} - wu\frac{dr}{ds} = 0. \tag{21}$$

The system of equations (21) is parabolic and describes the flow of a thin liquid layer under the action of inertial and pressure forces. Equations of the type (21) are encountered in the theory of a thin shock layer in the flow of an inviscid hypersonic stream over a blunt body [3, 4]. To obtain the solutions of these equations, one must assign the boundary and initial conditions: nonpenetration at the surface over which the flow occurs and the dimensionless pressure distribution $p_0(s)$ at the free surface; the initial data are obtained by joining the asymptotic expansions (12) or (15) for the flow retion II and the asymptotic expansions (20) for the flow region III. Thus, as the initial data, we obtain the profiles u(n) and w(n),

$$\lim_{s_3 \to 0} u_3(s_3, n_3) = \lim_{s_i \to \infty} u_i(s_i, n_i), \qquad \lim_{s_3 \to 0} w_3(s_3, n_3) = \lim_{s_i \to \infty} w_i(s_i, n_i), \tag{22}$$

where i = 1 or 2.

It is convenient to solve the system of equations (21), with the indicated boundary and initial conditions (22), in Mises variables (s, ψ), where ψ is the stream function. As a result, one can obtain the following solution:

$$w = C(\psi)/r, \; u = \sqrt{2D(\psi) - w^2}, \qquad \lambda \int_0^1 (u/Rr + w^2\cos\theta/ru)\, d\psi = \Delta p. \tag{23}$$

Here the functions $C(\psi)$ and $D(\psi)$ are found from the conditions (22).

If the equation of the sprayer surface along with the liquid film moves is assigned, then the solutions (23) enable one to determine the quantity p_w, the pressure at the surface of the sprayer. At those points where p_w proves to be less than the dimensionless pressure of saturated vapor of the liquid, the probability of detachment of the film from the sprayer surface is very high.

3. The solutions (23) can be used to construct the outer surface of a centrifugal sprayer which is attached for a given regime. If the distribution $\Delta p(s)$ is assigned and the initial profiles $u(\psi)$ and $w(\psi)$ at the exit from the flow region II are found, then the outer surface of the sprayer can be calculated.

Let us give an example of such a calculation. We assume that the sprayer surface in region II is known and the flow is attached here. For this it is sufficient that the sprayer surface be smooth in this region and the longitudinal size be $l_0 \gg \delta$. Also let $u(\psi)$ = const and $w(\psi)$ = const.

The initial data for the flow region III can be obtained without solving the problem for region II. For this it is sufficient to use the conservation laws operating in swirled streams [1, 2]. Because of the smallness of the film thickness, $v \ll u \sim w$, and we can write

$$\rho(u^2 + w^2) = 2\Delta p, \ w = C/r. \tag{24}$$

The variation of the external pressure in the vicinity of the corner point (region II) is small, from which we obtain, for the characteristic velocity in region III,

$$w_0 = \sqrt{2\Delta p/\rho}. \tag{25}$$

Since $u \ll w$ in regions I and II, the initial data for region III take the form

$$u = 0, \ (w/w_0) = 1 \ \text{for} \ (r/r_0) = 1. \tag{26}$$

Equation (24) in dimensionless variables is

$$u^2 + w^2 = 1; \ w = 1/r. \tag{27}$$

The parameter λ as a function of the liquid flow rate $\dot{m}$ (g/sec) in the sprayer is described by the relation

$$\lambda^{-1} = (p_0/\rho w_0^2)(2\pi r_0^2 \rho w_0/\dot{m})^{2/3}.$$

Here p_0 is the characteristic gas pressure at the outer boundary of the liquid film.

Equations (23) can be calculated easily for the initial conditions (26). In the vicinity of $r = 1$, their solution has the form

$$y = \frac{\sqrt{2}}{3\lambda}(1 - p_{01})(r-1)^{3/2} + \ldots,$$
$$\frac{dy}{dr} = \frac{\sqrt{2}}{2\lambda}(1 - p_{01})(r-1)^{1/2} + \ldots. \tag{28}$$

Here p_{01} is the ratio of the pressure of saturated vapor of the liquid to the characteristic pressure p_0.

The solution (28) is valid in the vicinity of $r = 1$; it shows that the radius of curvature of the stream in this region tends toward infinity, i.e., the surface of the sprayer in the vicinity of $r = 1$ must necessarily be flat.

An example of a calculation of the outer surface of a sprayer with $r = 0.3$ cm is given in Fig. 4. In the calculations it was assumed that the pressure distribution at the outer boundary of the liquid film depends on the angle of inclination of its surface to the axis of symmetry in accordance with $\sin^2 \theta$, while $p_{01} = 0.01$

NOTATION

y, axis of symmetry of the sprayer; r, distance from the axis of symmetry; δ, thickness of liquid film; s, coordinate measured along the surface of the sprayer; n, coordinate measured along a normal to the sprayer surface; φ, azimuthal coordinate; u, v, w, corresponding components of the velocity vector; p, pressure; F, centrifugal force; R, radius of curvature of the sprayer surface; $\Delta\Sigma$, element of the sprayer surface; θ, angle of inclination of the surface to the y axis; ρ, liquid density; l, L, characteristic lengths; h, Lamé coefficient; Eu, Euler number; A, α, λ, similarity parameters; ψ, stream function.

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EFFECTIVE TRANSPORT COEFFICIENTS IN A DISPERSE MEDIUM WITH ELLIPSOIDAL INCLUSIONS

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Expressions are obtained for the steady-state conductivity tensor for moderately concentrated heterogeneous materials with ellipsoidal inclusions.

If the linear dimensions of the mean temperature or concentration fields in a heterogeneous medium (consisting of a homogeneous matrix with discrete inclusions distributed in it) are significantly larger than the characteristic dimensions of the inclusions, then heat or mass transport is naturally described in terms of the continuum approximation. In this case it is sufficient to introduce effective thermal conductivities or diffusion coefficients for the medium as a whole [1, 2].

The determination of these effective coefficients for a medium with spherical inclusions has been considered in a number of papers, but the number of papers devoted to the analogous problem for a medium with nonspherical inclusions is quite small. A dilute dispersion of nonspherical inclusions was considered in [3]. A moderately concentrated dispersion of spheroidal inclusions was studied in [4, 5] in the dipole approximation (where the contribution of each inclusion to the mean field is replaced by that of a point dipole at the center of the given inclusion). In the present paper the general methods of [2] are used to analyze the properties of a heterogeneous material with ellipsoidal inclusions. The spatial distribution of the ellipsoids is assumed to be random and their orientation is assumed to obey a given statistical distribution law which is identical for all points of space. Then the material is macroscopically homogeneous, although it is not necessarily isotropic. We note that this theory is important not only in the description of materials with inclusions, but also as a model for the analysis of transport processes in isotropic and anisotropic polycrystalline media of more complicated structure [6, 7].

Statement of the Problem. In an anisotropic heterogeneous medium the relation between the mean heat flux and the gradient of the mean temperature has the form

$$\mathbf{q} = -\boldsymbol{\lambda}\cdot\nabla\tau, \tag{1}$$

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